Coding and Analysis for Sparse Communication

Massachusetts Institute of Technology

Student: Da Wang **Advisor:** Gregory Wornell

Signals, Information and Algorithms Laboratory, Research Laboratory of Electronics

rLe AT MIT

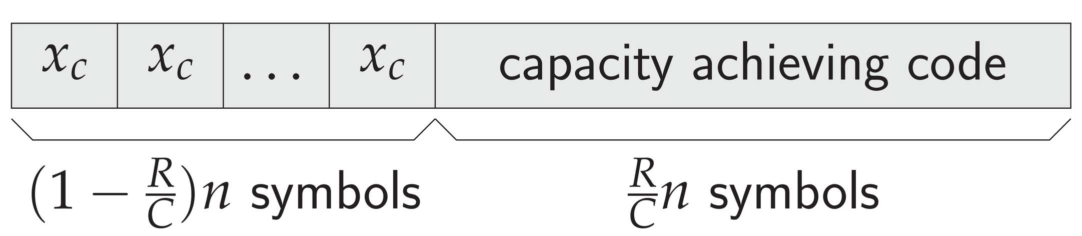
Motivation

Stages in Communication

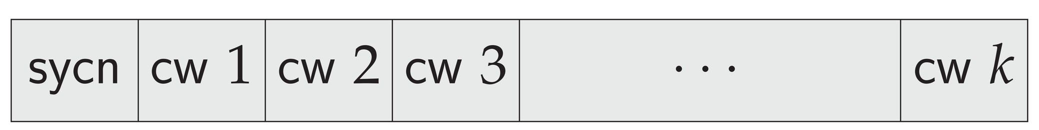
❶ **Synchronization**: detect the presence of a message and locate it.
❷ **Coding**: encode messages to transmit reliably over noisy channels.

Traditional Communication

- **Training**: separate synchronization and coding.

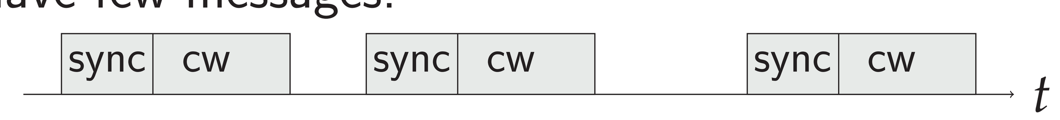


- The cost of synchronization is relatively low.



Sparse Communication

A type of communication that most transmissions have few messages.



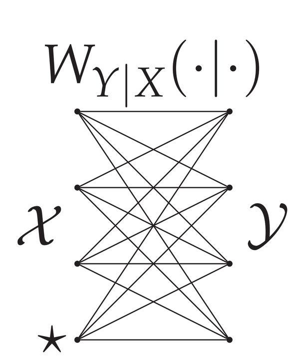
The cost of synchronization is no longer negligible!

How to communicate efficiently?

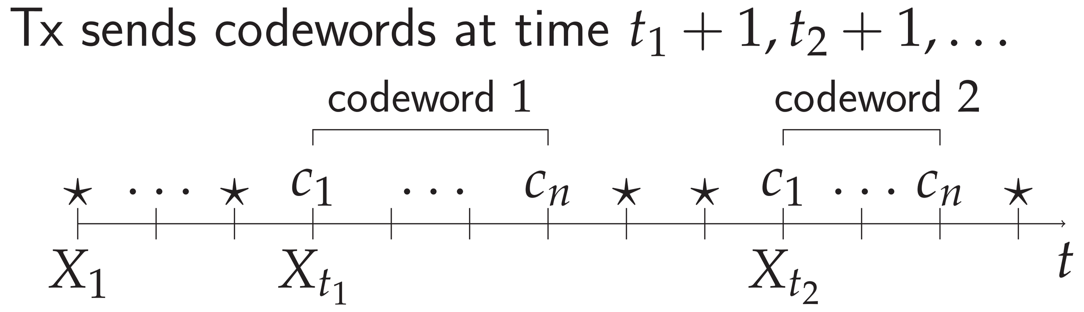
*Design codes for both **synchronization** and **information transmission**.*

Problem Formulation

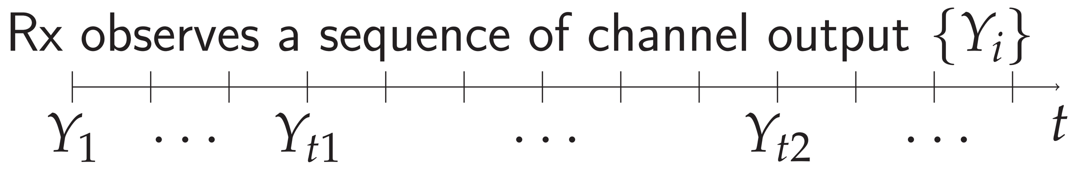
Asynchronous channel model



- $\star$: input symbol to model that nothing is sent.
- Tx sends codewords at time $t_1+1, t_2+1, \ldots$

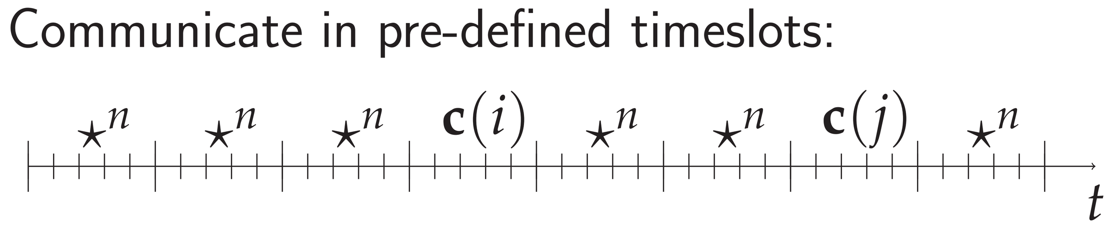


- Rx observes a sequence of channel output $\{Y_i\}$



Slotted simplification

Communicate in pre-defined timeslots:



synchronization $\begin{cases}\text{detect} \\ \text{locate}\end{cases}$ $\xrightarrow{\text{slotting}}$ detection only

Mathematical Setup

For a channel code $\mathcal{C} = \{X^n(k)\}$ with rate R, we have the following hypothesis testing problem:

$$\begin{cases} H_0: & Y_i \overset{i.i.d}{\sim} W(\cdot|\star) \quad i = 1, 2, \cdots n \\ H_1: & Y^n \sim W(\cdot|X^n(k)) \quad k \in \{1, 2, \cdots, M\} \end{cases}$$

Define error events

$$\mathcal{E}_{\text{Miss}} = \{H_1 \to H_0\}$$
$$\mathcal{E}_{\text{False alarm}} = \{H_0 \to H_1\}$$

and

$$P_m \triangleq \mathbb{P}[\mathcal{E}_{\text{Miss}}] \doteq \exp(-nE_m)$$
$$P_f \triangleq \mathbb{P}[\mathcal{E}_{\text{False alarm}}] \doteq \exp\left(-nE_f\right)$$

Intuition

- We want the channel outputs corresponding to codewords be "different" from noise.
- $E_m(R)$ and $E_f(R)$ indicate how different they can be.

Analysis objectives

Characterize the $E_m - E_f$ trade-off at rate R.

Special case: $E_m = 0$

Optimal $E_f(R)$ when $E_m = 0$

Given $E_m = 0$ and hence $P_m \to 0$,

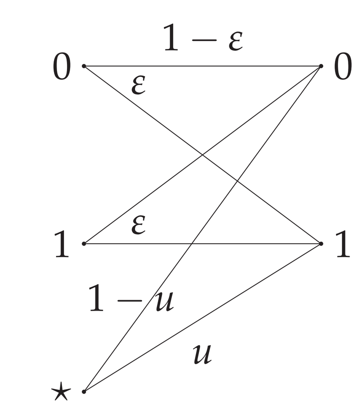
$$E_f(R) = \max_{P_X: I(P_X, W) = R} D(P_Y \| Q_\star)$$

- i.i.d codebook with distribution P_X
  - codeword output distribution P_Y
- noise output distribution $Q_\star = W(\cdot|\star)$.

✓ use rate-achieving i.i.d codebook rather than capacity-achiving codebook.

BSC Example

For a binary symmetric channel (BSC) with crossover probability ε and $W(\cdot|\star) = \text{Bernoulli}(u)$ $(u \leq 0.5)$, we can achieve



$$E_f(R) = D(\underbrace{\text{Bernoulli}(s^*)}_{\text{c.w. output dist.}} \| \underbrace{\text{Bernoulli}(u)}_{\text{noise output dist.}})$$

where $s^* \geq 0.5$ satisfies $H_b(s^*) - H_b(\varepsilon) = R$.

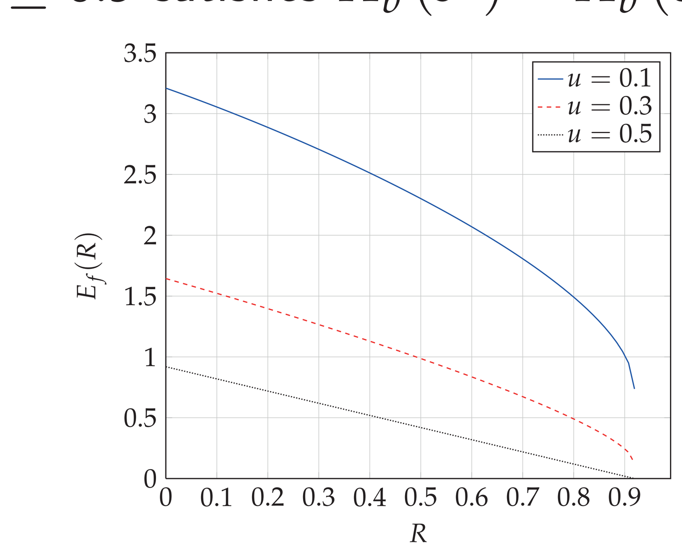
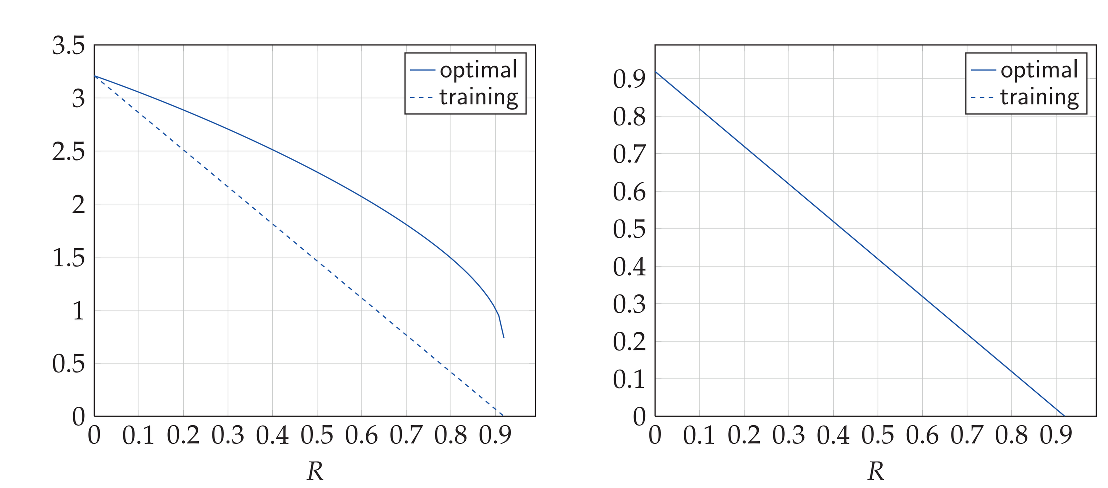


Figure: $E_f(R)$ for BSC with $\varepsilon = 0.01$

Comparison with training



(a) $u = 0.1$ (b) $u = 0.5$

Figure: Comparison with training for BSC with $\varepsilon = 0.01$.

✓ *Large gain over training at high rate.*

General case: $E_m \geq 0$

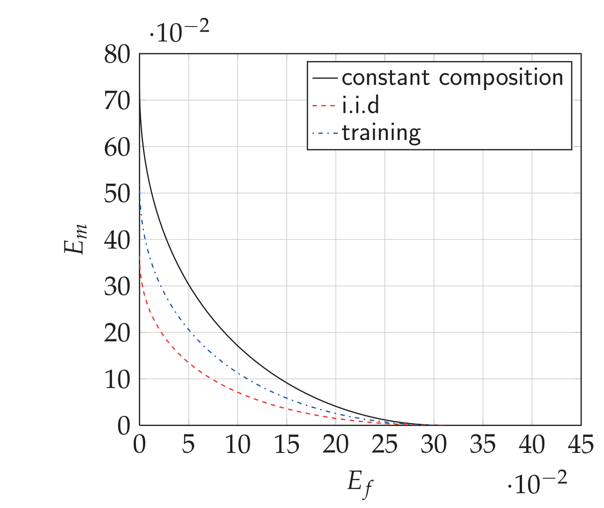
Achievable $E_f(R)$ when $E_m \geq 0$

Given $P_m \dot{\leq} \exp(-nE_m)$,

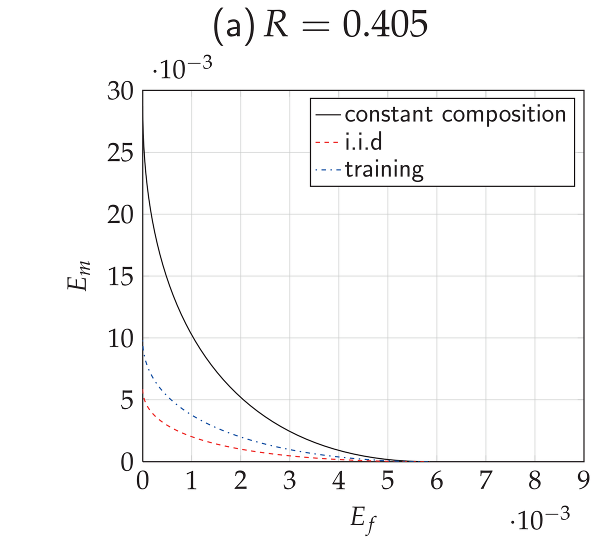
$$E_f(R, E_m) = \max_{P_X: I(P_X, W) \geq R} \ \min_{V: D(V\|W|P_X) \leq E_m} \left[D(Q_V \| Q_\star) + \{I(P_X, V) - R\}^+\right]$$

- achieved by constant composition codebook with maximizing distribution P_X^*.
- i.i.d codebook is suboptimal in general.
- non-trivial converse for DMC is difficult.

Comparison with i.i.d codebook and training for BSC channel



(a) $R = 0.405$



(b) $R = 0.708$

Figure: Performance comparison between constant composition codebook, i.i.d codebook, and training for BSC with $\varepsilon = 0.05$ and $u = 0.5$.

✓ *Again, large gain over training at high rate.*

Conclusion

For sparse communication, designing codes for both **detection** and **information transmission** jointly achieves significantly larger detection error exponents than the traditional separate sync–coding approach.